

# Quantum Noise in Conventional Optical Heterodyne Devices

Dechao He, Boya Xie, Yu Xiao, and Sheng Feng\*

*MOE Key Laboratory of Fundamental Quantities Measurement, School of Physics,  
Huazhong University of Science and Technology, Wuhan 430074, China*

(Dated: January 22, 2013)

By invoking the quantum theory of optical coherence, we theoretically show that the quantum noise in conventional optical heterodyne devices, which were previously identified as usual phase-insensitive amplifiers with additional quantum noise, is similar to that in optical homodyne devices, as verified by experimental data. Albeit more study is demanded to understand this result, it is certain that neither the uncertainty principle nor Caves's theorem for quantum noise of linear amplifiers sets a limit to the quantum noise of heterodyne devices.

PACS numbers: 42.50.Lc, 42.50.Dv, 42.50.Xa, 42.79.Sz

According to the profound work by Caves [1], linear amplifiers are classified into two types: Phase-insensitive amplifiers (PIA's) [2–6] and phase-sensitive amplifiers (PSA's) [7–11], both of which have received intense attention [12–15]. While a PSA is quantum-mechanically noiseless [1], all known PIA's are usually expected to add noise at least as large as the half-quantum of zero-point fluctuations, referred to the input, due to their internal vacuum-state modes. Conventional optical heterodyne devices were previously considered as usual PIA's [1, 16, 17]. They were expected to produce extra unavoidable quantum noise in amplified signals, to meet the requirement of the uncertainty principle [12, 13, 18]. On the other hand, optical homodyne devices were identified as PSA's [1]. In this Letter, we present a theoretical work showing that a conventional optical heterodyne device can be operated without additional quantum noise in comparison with a homodyne device, followed by confirmative experimental results.

First of all, the noise performance of optical heterodyne devices can be released from the constraint by the uncertainty principle, as shown below: At the output of a heterodyne device, one receives an electrical signal at the heterodyning frequency proportional to cross terms in light intensity:  $\hat{I}(t) \propto \hat{E}_1 \cos(\Omega t) + \hat{E}_2 \sin(\Omega t)$  ( $\hat{E}_{1,2}$  are the inphase and quadrature amplitudes of the optical signal being measured, respectively, and  $\Omega$  stands for the heterodyning frequency). According to previous works [12, 17, 19], one can split the electrical signal into two, with each being utilized to measure either the “cos” term or the “sin” term, which entails simultaneous measurement of  $\hat{E}_1$  and  $\hat{E}_2$ . Quantum mechanically,  $\hat{E}_{1,2}$  are two noncommuting variables and the uncertainty principle allows none to simultaneously measure them without paying any price. It was then argued that a 3dB-noise penalty must occur in optical heterodyning [12, 17, 19]. If  $\hat{E}_{1,2}$  were slowly-varying variables, it would be possible to de-couple them with narrow-band filters and a simultaneous measurement might be successfully carried out. However, when considering quantum noises, one should not expect  $\hat{E}_{1,2}$  to be slowly varying, as in the case in

optical heterodyning where both  $\hat{E}_{1,2}$  are responsible for white noise. Therefore, simultaneous measurement of the inphase amplitude and phase quadrature is impossible in optical heterodyning and the uncertainty principle does not force a heterodyne device to add quantum noise.

Secondly, one will see that the quantum noise in an optical heterodyne device is beyond the scope of Caves's theory [1], which, as a consequence, cannot require a heterodyne device to produce additional quantum noise either. One should note that, according Caves's theory, operation of a linear amplifier is governed by some linear evolution equations (Eqs. (2.5) in [1]), which should be understood as a generic solution to the Heisenberg equation of motion for the amplifier. According to the quantum theory of measurement, the Heisenberg equation (or the Schrödinger equation in an equivalent picture) stops playing its role when a quantum object is measured and a qualitative description of quantum measurement was first formulated as a postulate by von Neumann [20]. Similarly, the linear evolution equations in Caves's theory are not suitable for describing the physical process in optical heterodyning, wherein light intensity, which is proportional to photon number, is measured through photon absorption that is accompanied by state reduction for light. To quantitatively predict the quantum noise in optical heterodyning, one needs to resort to the quantum theory of optical coherence [21, 22], with which we will show below that the noise performance of optical heterodyne devices is similar to that of homodyne ones.

Before doing this, let first review some past works, the first one of which was reported by Oliver who studied the noise in optical heterodyning in early 1960's with a conclusion that the SNR in this process is twice as large as that in optical homodyning [16]. Later, Haus and Townes came up with a similar conclusion by connecting Oliver's treatment to the uncertainty principle [17]. However, both Oliver and Haus carried out their calculations based on classical physics. In 1970, Personick presented an image-band interpretation of the optical heterodyne noise [23], arguing that an image-band mode involved in optical heterodyning makes it quantum-mechanically

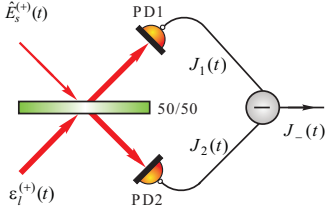


FIG. 1. (color online) Theoretical model for optical heterodyning (homodyning). At a balanced (50/50) beamsplitter, a signal carried by light in a coherent state is mixed with a strong coherent light, which serves as the local oscillator in the scheme. The light at each of the output ports of the beamsplitter is collected by a photodetector and the differenced photocurrents  $\hat{J}_-(t) \equiv J_1(t) - J_2(t)$  are fed into a spectrum analyzer for data processing. The signal field  $\hat{E}_s^{(+)}(t)$  is excited at optical frequency  $\omega_s$  and the local oscillator at  $\omega_l$ . The oscillator field is considered as a classical one [25–27] with its quantum noise neglected. The scheme is referred to optical heterodyning (homodyning) when  $\omega_s \neq \omega_l$  ( $\omega_s = \omega_l$ ).

twice noisier than homodyning. Since then, the concept of image-band mode was widely used to treat theoretical problems in optical heterodyning for fundamental research [12, 18], quantum information processing [24], and communications [13, 14, 19].

An obvious problem in Personick’s image-band interpretation was that he overlooked other vacuum modes at optical frequencies in the vicinity of  $\omega_i$ . If an image-band mode beats with the local oscillator producing 3dB additional quantum noise, other unexcited modes near the image-band mode should be able to do the same work. If so, the quantum noise in optical heterodyning will be unbelievably high. To remedy this shortcoming, a narrow-band electrical filter was conceptually added in the balanced-heterodyne model [13, 14, 19, 25] to filter out all beatnotes except two of them, i.e., the one between signal and oscillator, and the one between image-band vacuum mode and oscillator. Unfortunately, even by introducing narrow-band filters, the problem is not solved yet and the reason is similar to what we argued previously: Quantum noise is white. Quantum noise generated by all vacuum modes should pass any filters, no matter how narrow their pass-bands are, and contaminate the heterodyning signal equally.

As a matter of fact, no experimental evidences exist showing any difference between optical heterodyning and optical homodyning in terms of their noise performance. We will prove this theoretically first and then present an experiment to confirm it.

We start off by calculating the auto-correlation function of the differenced-photocurrent fluctuations  $\Delta J_-(t)$ , where  $\Delta A \equiv A - \langle A \rangle_s$  ( $\langle \cdot \rangle_s$  stands for statistical averaging), produced by the two photodetectors (Fig. 1):

$$\langle \Delta J_-(t) \Delta J_-(t + \tau) \rangle_s$$

$$= \sum_{i=1,2} \langle \Delta J_i(t) \Delta J_i(t + \tau) \rangle_s - \sum_{i,j=1,2; i \neq j} \langle \Delta J_i(t) \Delta J_j(t + \tau) \rangle_s. \quad (1)$$

For both optical homodyning and optical heterodyning,  $\langle \Delta J_i(t) \Delta J_i(t + \tau) \rangle_s$  ( $i = 1, 2$ ) reads [28]

$$\begin{aligned} & \langle \Delta J_i(t) \Delta J_i(t + \tau) \rangle_s \quad (i = 1, 2) \\ &= \eta \int_0^\infty dt' \langle \hat{I}_i(t - t') \rangle j_i(t') j_i(t' + \tau) \\ &+ \eta^2 \iint_0^\infty dt' dt'' \lambda_i(t - t', \tau + t' - t'') j_i(t') j_i(t''), \end{aligned} \quad (2)$$

where we assumed that the two photodetectors are identical and characterized by the same parameter  $\eta$  for their response to incident light.  $\hat{I}_i(t)$  ( $i = 1, 2$ ) are the intensity operators of light incident into the detectors and  $j_i(t - t')$  ( $i = 1, 2$ ) the photoelectrical current pulses produced at time  $t$  due to photoelectron emissions at  $t'$  in the  $i$ th detectors for  $t > t'$  [29, 30]. And  $\lambda_i(t, \iota) = \langle \mathcal{T} : \Delta \hat{I}_i(t) \Delta \hat{I}_i(t + \iota) : \rangle$  ( $i = 1, 2$ ) are the auto-correlation functions of light-intensity fluctuations generated by the two detectors. Here the symbol  $\mathcal{T} ::$  means time- and normal-ordering of the field operators. The first term in Eq. (2) accounts for the shot noise of light, while the second one depends on the fluctuation nature of the light being measured. If the said 3dB additional noise is present in optical heterodyning, then the auto-correlation functions  $\lambda_i$  in Eq. (2) for heterodyning will be different compared with the functions for the case of optical homodyning.

Using the matrix for the balanced beamsplitter, one can easily find the light intensities at the two output ports with strong local-oscillator approximation

$$\begin{aligned} \hat{I}_{1,2}(t) &= (1/2) \{ \hat{E}_s^{(-)}(t) \hat{E}_s^{(+)}(t) + \mathcal{E}_l^{(-)}(t) \mathcal{E}_l^{(+)}(t) \\ &\pm i [\mathcal{E}_l^{(+)}(t) \hat{E}_s^{(-)}(t) - \mathcal{E}_l^{(-)}(t) \hat{E}_s^{(+)}(t)] \}, \end{aligned} \quad (3)$$

wherein  $\hat{E}_s^{(+)}(t)$  and  $\mathcal{E}_l^{(+)}(t)$  are respectively the input signal field to be measured and the classical local-oscillator field. Particularly, one should note that included in the signal field are multi-modes with all possible frequencies [22]:  $\hat{E}_s^{(+)}(t) = iL^{-3/2} \sum_k (\frac{1}{2} \hbar \omega_k) \hat{a}_k e^{i\mathbf{k} \cdot \mathbf{r} - i\omega_k t}$  ( $L^3$  is the volume where the field is quantized and  $\hat{a}_k$  the annihilation operator corresponding to wavevector  $\mathbf{k}$ ). This distinguishes our theoretical work from all previous ones regarding the quantum noise in optical heterodyning. For optical heterodyning (homodyning), the signal field  $\hat{E}_s^{(+)}(t)$  is excited at frequency  $\omega_k \neq \omega_l$  ( $\omega_k = \omega_l$ ). Following the approach of Ou, Hong and Mandel [29, 30], one obtains (for detailed calculations, refer to the appendix in [28] with the only difference being that the oscillator was bichromatic there)

$$\lambda_i(t, \iota) = (1/4) \times$$

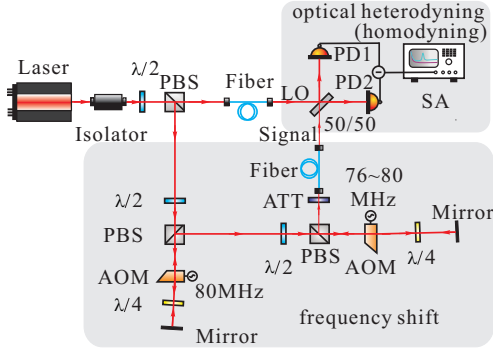


FIG. 2. (color online) Experimental schematics for optical heterodyning (homodyning). A continuous-wave single-frequency coherent light beam (spectral linewidth  $< 1$  kHz for 0.1s measurement time,  $\lambda = 1064$  nm) from a laser (Mephisto, Innolight GmbH) was split into two, one of which served as a local oscillator and the other was sent to two AOM's (Crystal Technology, LLC) for frequency shifting (upshift in one AOM and downshift in the other). The frequency-shifted beam was used as an input signal for heterodyning at a balanced (50/50) beamsplitter. Homodyning was realized when the net frequency shift by two AOM's was zero. Two photodiodes (ETX 500, JDS Uniphase) collected the light from the output ports of the beamsplitter and the differenced photocurrents were fed into a spectrum analyzer (N9320B, Agilent). The fibers that were used as spatial-mode cleaners were single-mode polarization-maintaining fibers.  $\lambda/2$ : Half-wave plate.  $\lambda/4$ : Quarter-wave plate. PBS: Polarizing beamsplitter. AOM: Acousto-optic modulator. LO: Local Oscillator. ATT: Optical-power attenuator. PD1 & PD2: Photodiodes plus preamplifiers. SA: Spectrum analyzer.

$$\begin{aligned}
 & [\mathcal{E}_l^{(+)}(t)\mathcal{E}_l^{(-)}(t+\iota) < \Delta\hat{E}_s^{(-)}(t)\Delta\hat{E}_s^{(+)}(t+\iota) > \\
 & + \mathcal{E}_l^{(-)}(t)\mathcal{E}_l^{(+)}(t+\iota) < \Delta\hat{E}_s^{(-)}(t+\iota)\Delta\hat{E}_s^{(+)}(t) > \\
 & - \mathcal{E}_l^{(+)}(t)\mathcal{E}_l^{(+)}(t+\iota) < \Delta\hat{E}_s^{(-)}(t)\Delta\hat{E}_s^{(-)}(t+\iota) > \\
 & - \mathcal{E}_l^{(-)}(t)\mathcal{E}_l^{(-)}(t+\iota) < \Delta\hat{E}_s^{(+)}(t+\iota)\Delta\hat{E}_s^{(+)}(t) >] \\
 & + O(\mathcal{E}). \quad (i = 1, 2) \quad (4)
 \end{aligned}$$

According to the definition of coherent states [22], the eigenstates of the field operator  $\hat{E}_s^{(+)}(t)$ , all the correlation functions of field fluctuations in the four leading terms in the above equation vanish, i.e.,  $\lambda_i(t, \iota) \approx 0$ , for both the case of heterodyning and homodyning. One must note that the aforesaid image-band mode was already included in  $\hat{E}_s^{(-)}(t)$  in the above calculations for optical heterodyning! Plugging Eqs. (3) and (4) into Eq. (2), one arrives at

$$< \Delta J_i(t) \Delta J_i(t + \tau) >_s \approx \frac{\eta \mathcal{E}_l^2}{2} \int_0^\infty dt' j_i(t') j_i(t' + \tau), \quad (5)$$

Following similar reasoning, one can show without much difficulty that  $< \Delta J_i(t) \Delta J_j(t + \tau) >_s \approx 0$  for  $i \neq j$  ( $i, j = 1, 2$ ). Therefore, Eq. (5) can be rewritten as, by setting

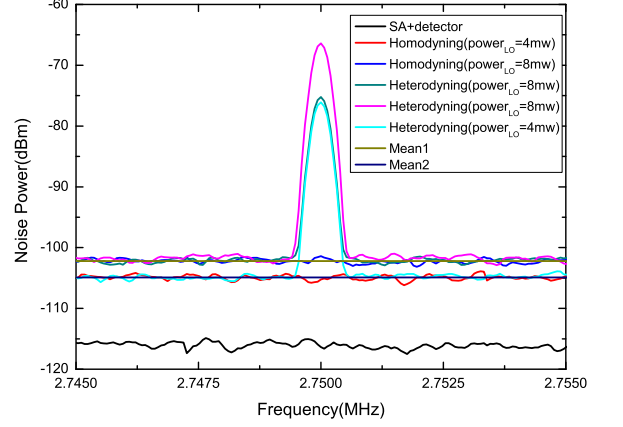


FIG. 3. (color online) Quantum-noise floors in optical heterodyning (homodyning). We identified the quantum-noise floors by two ways: (1) The noise levels upshifted by 3dB when the oscillator power was doubled (electronics noise subtracted) and (2) the noise levels agreed with theoretical values for quantum noise. RBW=0.3kHz and the oscillator powers were 4mW and 8mW. The power of the optical signal was tens of picowatts.

$$\tau = 0,$$

$$< \{\Delta[J_-(t)]\}^2 >_s \approx \frac{\eta \mathcal{E}_l^2}{2} \int_0^\infty dt' \sum_{i=1}^2 j_i^2(t'), \quad (6)$$

which is nothing but the total differenced-photocurrent fluctuations. Since the photoelectrical current pulses  $j_i(t')$  are determined only by the technical parameters of the photodiodes, the current fluctuations  $< \{\Delta[J_-(t)]\}^2 >_s$  take the same time-independent values for both homodyning and heterodyning. This formally proves our earlier statement that the said 3dB additional noise does not exist in optical heterodyning, whose noise performance is comparable to optical homodyning.

Although study of optical homodyning has proven that a vacuum mode can generate quantum noise, not available yet is a clear pictorial explanation why the image-band vacuum mode does not produce 3dB additional quantum noise in optical heterodyning. More theoretical development is surely demanded along this line. Whatever physics is hidden, experimental study must be carried out to test the above theoretical analysis, which drove us to preform an experiment with a result confirming that the noise performance of an optical heterodyne device is indeed comparable to that of a homodyne one (see Fig. 2). As the experimental data show (Fig. 3), the quantum-noise floors in optical heterodyning were at the same levels as in optical homodyning.

Regarding data interpretation, one should note the following two points: First, optical loss should be minimized and the photodiodes must collect as many pho-

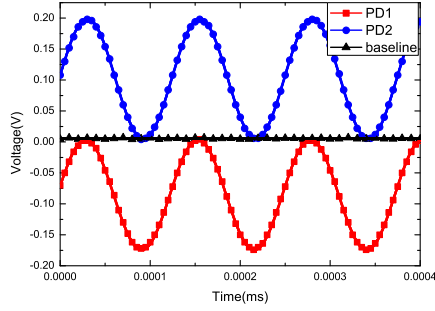


FIG. 4. (color online) Typical interference fringes in optical heterodyning when the light powers were equal at the beam-splitter input. Fringe visibility close to unity was observed by the two photodiodes. The baseline was obtained when no light was sent to the photodiodes.

tons as possible for high quantum efficiency. Secondly, the spatial mode-matching between the optical signal and the oscillator must be optimized for high fringe visibility. Optical loss, non-perfect quantum efficiency, and spatial mode-mismatching allow vacuum fluctuations to play roles in the detection and may prevent one from observing the aforementioned 3dB additional noise in optical heterodyning. Our photon collection efficiency was about 70% taking into account all optical loss (including the quantum efficiencies of the photodiodes, which were 75% and 76% respectively) and the fringe visibility was 98% and 99% seen by the photodiodes, respectively (Fig. 4). Accordingly, if the predicted 3dB additional noise was present in optical heterodyning, we should be able to observe a noise difference of 2.3dB between optical heterodyning and optical homodyning, which definitely did not show up in the data.

To conclude, we have shown that neither the uncertainty principle nor Caves's theory puts a limit on the noise performance of optical heterodyne devices. For coherent optical signal as input, we have calculated the variance of the photocurrents for optical heterodyning and optical homodyning, showing no difference between the two case as verified by experimental data. Further investigation is expected for us to fully understand the results.

This work was supported by the National Natural Science Foundation of China (grant No. 11174094) and Huazhong University of Science and Technology through the Startup Funding for New Faculty.

\* fengsf2a@gmail.com

- [1] C. M. Caves, Phys. Rev. D **26**, 1817 (1982).
- [2] M. W. P. Strandberg, Phys. Rev. **106**, 617 (1957).
- [3] W. H. Louisell, A. Yariv, and A. E. Siegman, Phys. Rev. **124**, 1646 (1961).
- [4] H. A. Haus and J. A. Mullen, Phys. Rev. **128**, 2407 (1962).
- [5] Z. Y. Ou, S. F. Pereira, and H. J. Kimble, Phys. Rev. Lett. **70**, 3239 (1993).
- [6] J. ichi Yoshikawa, Y. Miwa, R. Filip, and A. Furusawa, Phys. Rev. A **83**, 052307 (2011).
- [7] P. K. Lam, T. C. Ralph, E. H. Huntington, and H.-A. Bachor, Phys. Rev. Lett. **79**, 1471 (1997).
- [8] S.-K. Choi, M. Vasilyev, and P. Kumar, Phys. Rev. Lett. **83**, 1938 (1999).
- [9] V. Josse, M. Sabuncu, N. J. Cerf, G. Leuchs, and U. L. Andersen, Phys. Rev. Lett. **96**, 163602 (2006).
- [10] Z. Tong, C. Lundstrom, P. A. Andrekson, C. J. McKinstrie, M. Karlsson, D. J. Blessing, E. Tupsuwanakul, B. J. Puttnam, H. Toda, and L. Gruner-Nielsen, Nature Photonics **5**, 430 (2011).
- [11] N. V. Corzo, A. M. Marino, K. M. Jones, and P. D. Lett, Phys. Rev. Lett. **109**, 043602 (2012).
- [12] Y. Yamamoto and H. A. Haus, Rev. Mod. Phys. **58**, 1001 (1986).
- [13] C. M. Caves and P. D. Drummond, Rev. Mod. Phys. **66**, 481 (1994).
- [14] J. H. Shapiro, IEEE J. Sel. Top. Quantum Electron. **15**, 1547 (2009).
- [15] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).
- [16] B. M. Oliver, Proc. IRE **49**, 1960 (1961).
- [17] H. A. Haus and C. H. Townes, Proc. IRE **50**, 1544 (1962).
- [18] H. P. yuen, Phys. Lett. **91**, 101 (1982).
- [19] H. P. yuen and J. H. Shapiro, IEEE Trans. Inf. Theory **26**, 78 (1980).
- [20] V. B. Braginsky and F. Y. Khalili, *Quantum Measurement* (Cambridge University Press, 1992).
- [21] R. J. Glauber, Phys. Rev. **130**, 2529 (1963).
- [22] R. J. Glauber, Phys. Rev. **131**, 2766 (1963).
- [23] S. D. Personick, Bell Syst. Tech. J. **50**, 213 (1971).
- [24] C. Weedbrook, S. Pirandola, R. Garcia-Patron, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, Rev. Mod. Phys. **84**, 621 (2012).
- [25] H. P. Yuen and V. W. S. Chan, Opt. Lett. **8**, 177 (1983).
- [26] L. Mandel, Phys. Rev. Lett. **49**, 136 (1982).
- [27] B. L. schumaker, Opt. Lett. **5**, 189 (1984).
- [28] S. Feng, Z. H. Lu, J. Zhang, and C. G. Shao, e-print arXiv:1112.3155v2.
- [29] Z. Y. Ou, C. K. Hong, and L. Mandel, J. Opt. Soc. Am. B **4**, 1574 (1987).
- [30] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, 1st ed., Vol. 1 (Cambridge University Press, New York city, New York, 1995).